Deriving and Calculation of Values of Optimum Balance for Spherical Aberration from Five- -Order for Array of Obscured Circular Synthetic Apertures

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Abstract

In this research we derived the equation of optimum balance for spherical aberration from five order for array of obscured circular synthetic apertures, also to calculating the value of optimum balance for set from obscured circular synthetic apertures (N=1,2,6,8), the obscuration ratio (ε=0,0.25,0.5,0.75),by using Math LAB program, and derivative the equation of point spread function (PSF) for this case, by substituting the value of optimum balance in equation point spread function for spherical aberration from five order by using MathCAD program, the results show an increasing the values of spherical aberration coefficients from five order with increasing the apertures number, addition to increase the point spread function with increasing the obscuration ratio.

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Introduction

"Synthetic Aperture " can be defined as a structure to separate optical systems of large individual aperture function sometimes called " Mosaic" or "Segmented Mirror" . Synthetic aperture is an image system for independent optical systems which are together sharing the image domain [1].

Synthetic aperture techniques are commonly used to obtain high resolution from data acquired using low resolution sensors, these techniques are commonly used in modern sonar and radar systems, being designated by Synthetic Aperture Sonar (SAS) and by Synthetic Aperture Radar (SAR) systems, respectively; this kind of systems are presently used in civil and military applications [2].

One of the most important problems in the diffraction theory of aberrations is that of balancing the individual aberration coefficients in such a manner as to achieve optimum image quality [3,4,5].

The manner of the optimum balance was used by Marechal [6]. To find the minimum value for mean square deviation in wave front we get maximum value of central intensity within strehl criterion, so that the mean square deviation of wave front (variance σ^2) [3,7,8,9], can be obtained as:

$$V = \sigma^2 = \left[\overline{w^2}\right] - \left[\overline{w}\right]^2 = \frac{1}{A}\iint_{area} w^2 dA - \left[\frac{1}{A}\iint_{area} w dA\right]^2 \quad \text{.........(1)}$$

Where σ is the root-mean-square wave front error, which is defined as the mean of the squared wave front error minus the squared mean of the wave front error and A is the area of the whole optical pupil [3,7,8,9].

The mean square deviation wave front (variance) can be expressed by using polar coordinate[3,4,8].

$$V = \frac{1}{A}\iint_{r\phi} \left[w(r,\phi)\right]^2 r dr d\phi - \left[\frac{1}{A}\iint_{r\phi} w(r,\phi) r dr d\phi\right]^2 \quad \text{..........(2)}$$

"Barakat & Houston"[10] and "Rivolta" [11] work to solve the variance equation of obscured circular aperture by considering the complete circular aperture of the spatial case when $\varepsilon = 0$ [3].

In case of using an array of obscured circular synthetic apertures we have to make a modification for the variance equation to make it appropriate. To calculate the integration within the given area for array of obscured circular synthetic apertures, we write the variance equation for Marechal in the image plane [6]. as follows

$$V = \left[\frac{1}{\pi} \int_0^{2\pi} \int_0^{\frac{1}{\sqrt{N}}} [w(r,\phi)]^2 r dr d\phi - \frac{1}{\varepsilon^2 \pi} \int_0^{2\pi} \int_0^{\frac{\varepsilon}{\sqrt{N}}} [w(r,\phi)]^2 r dr d\phi \right]$$

$$- \left[\left[\frac{1}{\pi} \int_0^{2\pi} \int_0^{\frac{1}{\sqrt{N}}} w(r,\phi) r dr d\phi \right]^2 - \left[\frac{1}{\varepsilon^2 \pi} \int_0^{2\pi} \int_0^{\frac{\varepsilon}{\sqrt{N}}} w(r,\phi) r dr d\phi \right]^2 \right] \qquad \ldots\ldots\ldots\ldots\ldots(3)$$

where

$$w(r,\phi) = w_{20} r^2 + w_{40} r^4 + w_{60} r^6 + w_{80} r^8 + \ldots\ldots\ldots [5,12]$$

Deriving the Equation of Optimum Balance of Spherical Aberration of Five – Order for Array of Obscured Circular Synthetic Apertures

The spherical aberration of five order is given by relation

$$w = w_{20} r^2 + w_{40} r^4 + w_{60} r^6 \qquad \ldots\ldots\ldots\ldots\ldots(4)$$

Substitute equation (4) by equation (3), we obtain:

$$V = \left\{ \frac{1}{\pi} \int_0^{2\pi} \int_0^{\frac{1}{\sqrt{N}}} \left[w_{20} r^2 + w_{40} r^4 + w_{60} r^6 \right]^2 r dr d\phi - \frac{1}{\varepsilon^2 \pi} \int_0^{2\pi} \int_0^{\frac{\varepsilon}{\sqrt{N}}} \left[w_{20} r^2 + w_{40} r^4 + w_{60} r^6 \right]^2 r dr d\phi \right\}$$

$$- \left\{ \left[\frac{1}{\pi} \int_0^{2\pi} \int_0^{\frac{1}{\sqrt{N}}} (w_{20} r^2 + w_{40} r^4 + w_{60} r^6) r dr d\phi \right]^2 - \left[\frac{1}{\varepsilon^2 \pi} \int_0^{2\pi} \int_0^{\frac{\varepsilon}{\sqrt{N}}} (w_{20} r^2 + w_{40} r^4 + w_{60} r^6) r dr d\phi \right]^2 \right\}$$

$$\ldots\ldots\ldots\ldots(5)$$

When the aperture number equal one (N=1) and obscuration ratio equal zero (ε=0), we obtain

$$V = \frac{1}{12} w_{20}^2 + \frac{4}{45} w_{40}^2 + \frac{9}{112} w_{60}^2 + \frac{1}{6} w_{20} w_{40} + \frac{3}{20} w_{20} w_{60} + \frac{1}{6} w_{40} w_{60} \ldots\ldots\ldots\ldots(6)$$

By solving the equation (6), we obtain

$$V = \frac{w_{60}^2}{2800} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

Substitute the value of equation (7) in equation below

$$V \le \frac{\lambda^2}{180}$$

We obtain

$$\frac{w_{60}^2}{2800} \le \frac{\lambda^2}{180} \Rightarrow w_{60} = 3.944\lambda$$

$$\therefore w_{40} = -5.9\lambda$$

$$w_{20} = 2.36\lambda$$

For simplification the complex calculations, programming the equations by using MATLAB program, when the different apertures number and different obscuration ratio of optical system, we obtain the Table (1) which represents the value of optimum balance of spherical aberration of five order for array of obscured circular synthetic apertures.

Table (1) The Values of Optimum Balance of Spherical Aberration of Five Order .

N=1	ε= 0	ε =0.25	ε =0.75
W_{20}	2.3664 λ	1.3334 λ	0.1076 λ
W_{40}	-5.9161 λ	-2.3544 λ	0.3922 λ
W_{60}	3.9441 λ	1.1867 λ	0.2978 λ
N=2	ε= 0	ε =0.5	ε =0.75
W_{20}	3.5935 λ	2.2849 λ	0.1895 λ
W_{40}	-23.1008 λ	-10.7075 λ	-0.7238 λ
W_{60}	34.2234 λ	12.8238 λ	2.3397 λ
N=6	ε= 0	ε =0.25	ε =0.75
W_{20}	17.6629 λ	17.2127 λ	3.5867 λ
W_{40}	-350.4085 λ	-319.4164 λ	-46.8273 λ
W_{60}	1.5726 λ	1.3771 λ	154.038 λ
N=8	ε= 0	ε =0.5	ε =0.75
W_{20}	27.0750 λ	15.4749 λ	5.2690 λ
W_{40}	-717.8020 λ	-326.5043 λ	-93.6026 λ
W_{60}	4.2984λ	1.6798 λ	417.2879 λ

By inserting the value of optimum balance of spherical aberration from five order from Table (1) in equation the point spread function for array of obscured circular synthetic apertures [13].

$$PSF = \frac{1}{(\pi - \varepsilon^2 \pi)^2} \left| \begin{array}{l} \int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \cos\left\{2\pi[w_{20}(x^2+y^2)+w_{40}(x^2+y^2)^2+w_{60}(x^2+y^2)^3]+z'x'\right\} dxdy.\sum_{j=1}^{N} e^{iz'x_j} \\ -\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}} \cos\left\{2\pi[w_{20}(x'^2+y'^2)+w_{40}(x'^2+y'^2)^2+w_{60}(x'^2+y'^2)^3]+z'x'\right\} dx'dy'.\sum_{j=1}^{N} e^{iz'x_j} \\ +\int_{-\frac{1}{\sqrt{N}}}^{\frac{1}{\sqrt{N}}} \int_{-\sqrt{\frac{1}{N}-y^2}}^{\sqrt{\frac{1}{N}-y^2}} \sin\left\{2\pi[w_{20}(x^2+y^2)+w_{40}(x^2+y^2)^2+w_{60}(x^2+y^2)^3]+z'x'\right\} dxdy.\sum_{j=1}^{N} e^{iz'x_j} \\ -\int_{-\frac{\varepsilon}{\sqrt{N}}}^{\frac{\varepsilon}{\sqrt{N}}} \int_{-\sqrt{\frac{\varepsilon^2}{N}-y^2}}^{\sqrt{\frac{\varepsilon^2}{N}-y^2}} \sin\left\{2\pi w_{20}(x'^2+y'^2)+w_{40}(x'^2+y'^2)^2+w_{60}(x'^2+y'^2)^3]+z'x'\right\} dx'dy'.\sum_{j=1}^{N} e^{iz'x_j} \end{array} \right|^2$$

…………(12)

Equation (12) represents the point spread function for array of obscured circular synthetic apertures with spherical aberration of five order.

By using MathCAD program we calculated the point spread function (PSF) for array of obscured circular synthetic apertures with inserting the value of optimum balance of spherical aberration from five orders by different obscuration ratios (ε=0, 0.25, 0.5, 0.75) and different values of the apertures number (N=1, 2, 6, 8).

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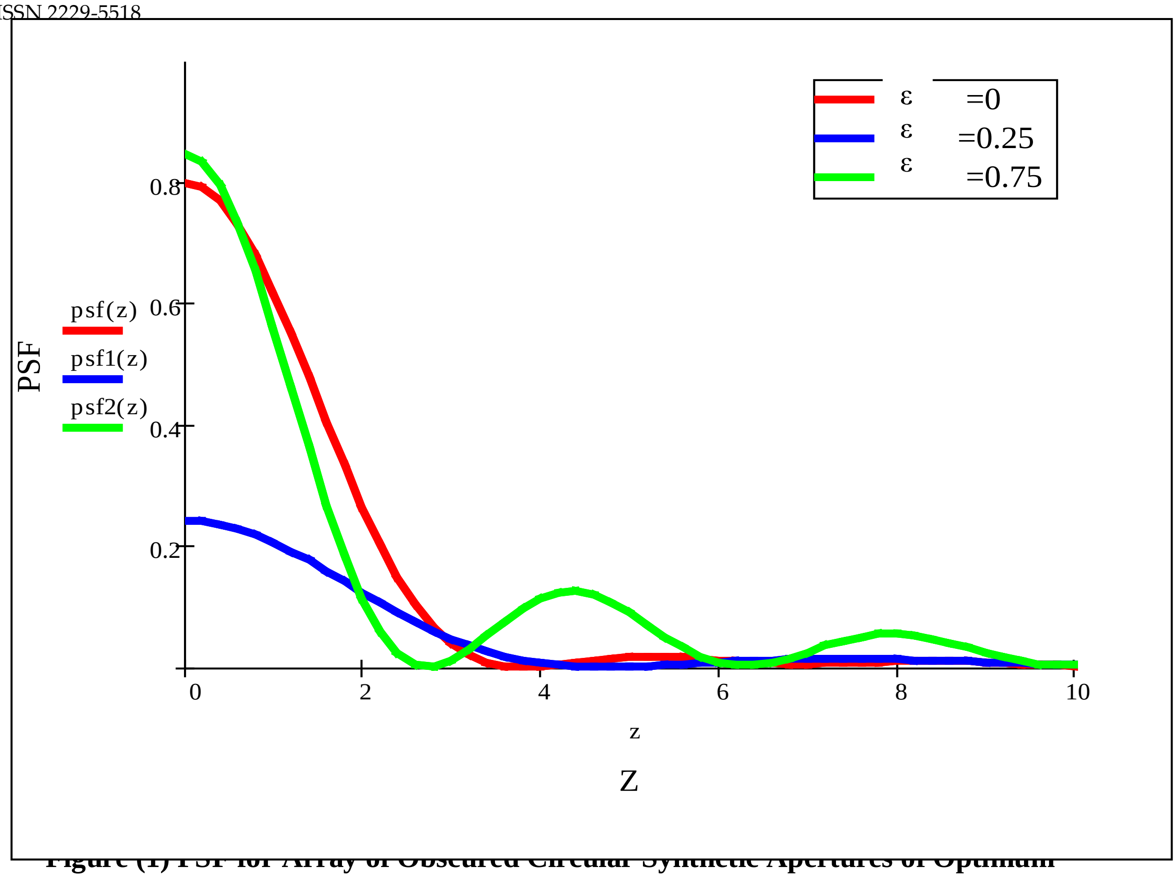

Figure (1) PSF for Array of Obscured Circular Synthetic Apertures of Optimum Balance of Spherical Aberration from Five Order when (N=1)

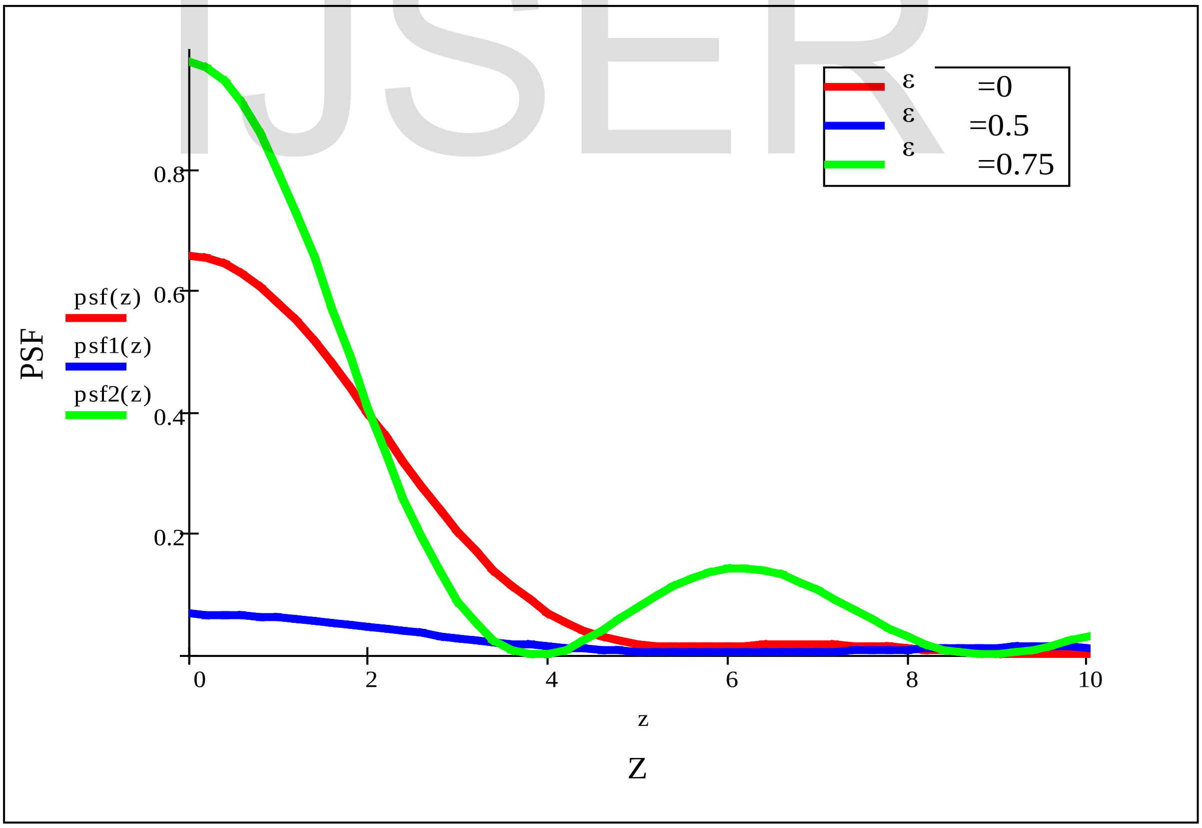

Figure (2) PSF for Array of Obscured Circular Synthetic Apertures of Optimum Balance of Spherical Aberration from Five Order when (N=2)

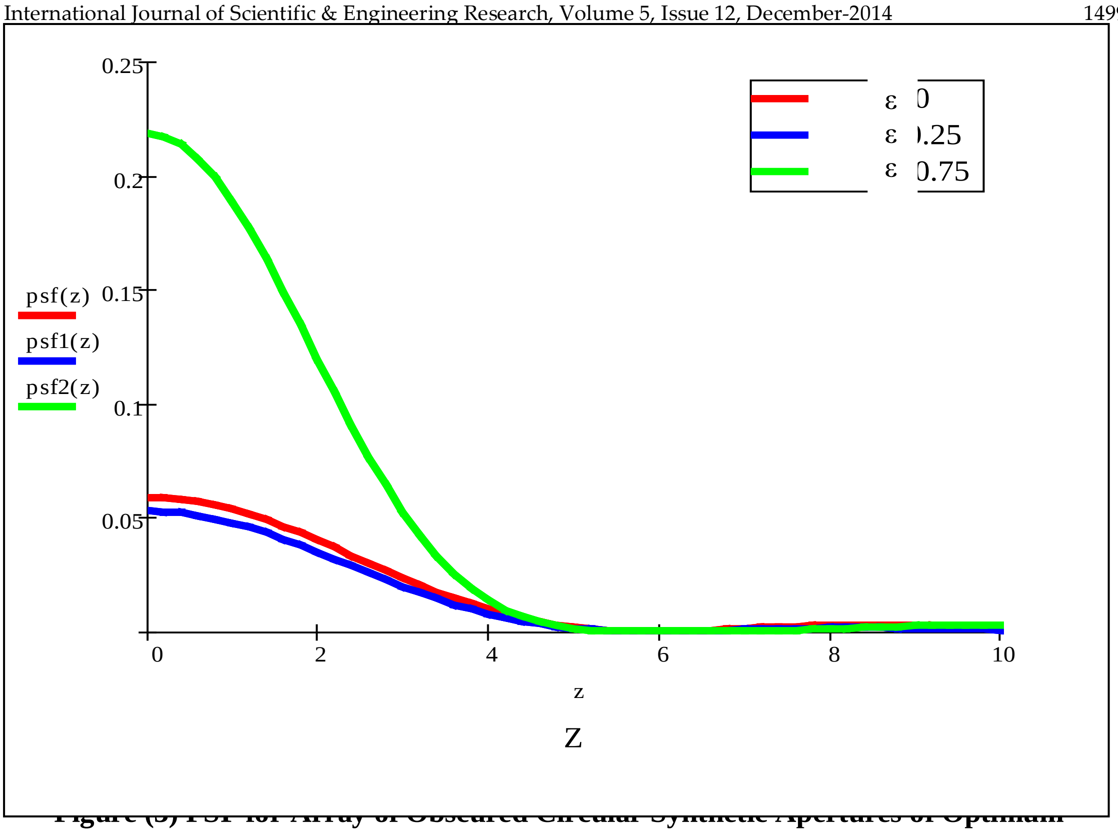

Figure (3) PSF for Array of Obscured Circular Synthetic Apertures of Optimum Balance of Spherical Aberration from Five Order when (N=6)

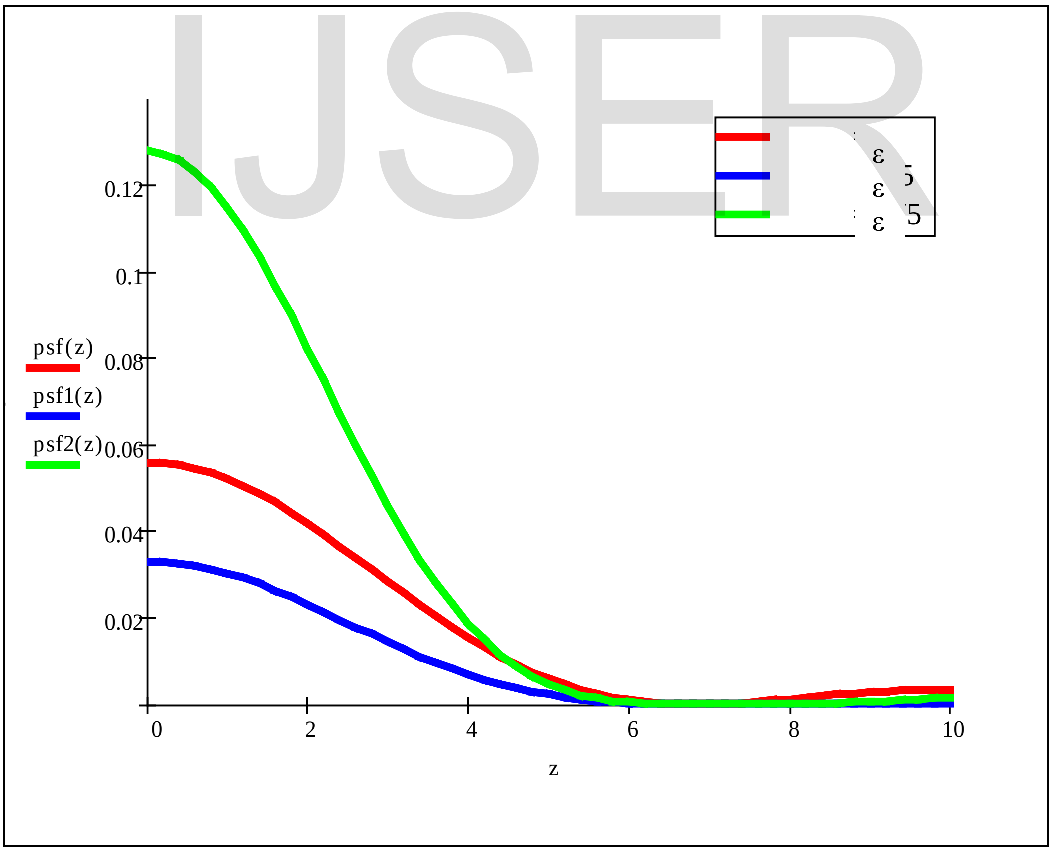

Figure (4) PSF for Array of Obscured Circular Synthetic Apertures of Optimum Balance of Spherical Aberration from Five Order when (N=8)

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Discussion

The spherical aberration is important because of the single aberration effect when the object is situated on the axis optical system; therefore, we derived the special relation with the optimization case for this aberration depending on the Strehl principle and Marechal relation.

The aim of using the obscured apertures is to make images better and reduce the effect of aberration.

When solving the equation (12) in MathCAD program and with inserting the values of optimum balance used in the is research according to the Table (1) by using Math LAB program and drawing the curves of PSF on the vertical axis and (Z) on the horizontal axis, we get

Figures (1),(2),(3) and (4) represent the intensity distribution curves of point spread function for individual aperture and synthetic apertures (N=2,6 and 8) with different values of obscuration ratios (ε=0,0.25,0.5 and 0.75) by using the values of optimum balance with the existence of spherical aberration of five order according the Table (1), we found that:

We notice from the Figures and Table (1) to increase values of the apertures number (N) is equivalent to increase in values of coefficients w_{20}, w_{40} and w_{60} .

Because the values of optimum balance (which was derived in this research) are equivalent to the focus error which is intrinsic in the lens or (optical lens system) and it is also in opposite direction to the intrinsic focus error, this the values (derived in this research) removed the intrinsic focus error and make improvement in the optical system. So the optical system becomes free of focus error, and this result appears clearly in the above Figures. We can determine the physical meaning of the state as " we know the aberration is incapability the wave to keep its spherical shape which means deviation the spherical shape of wave from the ideal state to another shape and when we applied another deviation for this wave in an opposite direction this leads to correcting the shape to the ideal shape as a result " .

We notice increasing the value of (PSF) with increasing of obscuration ratio (ε): therefore, improving the optical system contains the amounts from the aberration from during increase the value (ε).

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